Health care costs continue to rise at unprecedented rates in the United States. While past research has addressed optimal patient scheduling to improve the utilization of operating theater resources (physicians, specialized equipment, and operating rooms), little research has attempted to optimize material requirements and safety stocks to support these optimized schedules. Although classical inventory models can and should be used, additional considerations in hospital environments remain unaddressed. Based on our empirical observations, significant opportunities exist to reduce waste by improving the planning and coordination of materials and information in hospital operating theaters. To quantify these opportunities, we develop an analytic optimization framework for planning and executing material quantity preparation and placement decisions to support hospital operating theaters. Our model considers the non-stationary stochastic demand associated with block scheduling of surgical departments and a stochastic bill of materials, common in surgical treatments. Our approach is novel in that we formulate the stochastic decision problem exactly as a linear program by leveraging the convexity of the expected cost functions. The model is computationally efficient and flexible, allowing fast solutions and unique considerations for individual hospitals while considering uncertainty explicitly. We conduct a numerical study and sensitivity analysis to pinpoint the leverage points in the system for reducing inventory carrying and materials management costs. Surprisingly, reducing the uncertainty in the physician-determined bill of materials through standardization exhibited the greatest potential for savings.

Keywords: Health care, inventory theory, hospital materials management, multi-echelon inventory optimization, stochastic nonstationary demand, stochastic bill of materials

Introduction

Health care costs are rising at unprecedented rates in the United States and are higher than health care costs in most other developed countries. For example, Leonhardt (2006) notes that, while Greek and Canadian citizens spend an average of US $2,300 and US $3,300 annually per capita on health care, respectively. U.S. citizens spend more than US $6,000 per capita. Moreover, this is occurring as baby boomers are entering the most medically expensive years of their lives. The Boston Municipal Research Bureau (BMRB, 2006) finds that health care costs of the City of Boston have doubled from 2000 to 2006. Despite rapidly increasing costs, the true health care costs continue to rise at unprecedented rates in the United States. While past research has addressed optimal patient scheduling to improve the utilization of operating theater resources (physicians, specialized equipment, and operating rooms), little research has attempted to optimize material requirements and safety stocks to support these optimized schedules. Although classical inventory models can and should be used, additional considerations in hospital environments remain unaddressed. Based on our empirical observations, significant opportunities exist to reduce waste by improving the planning and coordination of materials and information in hospital operating theaters. To quantify these opportunities, we develop an analytic optimization framework for planning and executing material quantity preparation and placement decisions to support hospital operating theaters. Our model considers the non-stationary stochastic demand associated with block scheduling of surgical departments and a stochastic bill of materials, common in surgical treatments. Our approach is novel in that we formulate the stochastic decision problem exactly as a linear program by leveraging the convexity of the expected cost functions. The model is computationally efficient and flexible, allowing fast solutions and unique considerations for individual hospitals while considering uncertainty explicitly. We conduct a numerical study and sensitivity analysis to pinpoint the leverage points in the system for reducing inventory carrying and materials management costs. Surprisingly, reducing the uncertainty in the physician-determined bill of materials through standardization exhibited the greatest potential for savings.

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Introduction

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care cost structure remains enigmatic and has been shown to translate into wildly varying patient charges for similar treatments (Dorschner, 2006).

There is evidence, however, that certain costs are rising faster than others. Between 2000 and 2002, Solucient (2004) reports that the fastest growing average annual hospital cost rates are in diagnostic imaging (36%), operating rooms (32%), intensive care (27%), medical supplies (26%), and pharmaceuticals (22%). Taheri et al. (2000) assess which costs are under the influence of physicians. They categorize health care costs into three types: variable, fixed, and indirect. In their detailed analysis of a trauma center, they find that 35% of the cost is variable, such as supplies, and under the influence of the physician.

Based on our observations, there is a high degree of disorganization and lack of standard processes in the area of materials management. In some hospitals, nurses are responsible for materials planning, and in others, pharmacists are responsible. We observed a propensity for physicians to requisition materials “just in case” and not necessarily due to patient need. Often, there is unused (but still usable) material at the end of treatment. In our research, unused materials were never restocked in a hospital’s central storage area. Although physicians have much influence over variable costs, in our observation they are not necessarily provided with ample information, economics, organization, or incentives to focus on the efficient use of materials.

These observations are echoed by other researchers. In U.S. hospitals, Jarett (1998) estimates that inventory costs often represent between 10 and 18% of a hospital’s revenues. In European hospitals, Beaulieu and Landry (2002) show that hospital inventory costs can represent between 30 and 40% of a hospital’s annual budget. The Center for Health Design (Brannen, 2006) identifies unused surgical supplies as a significant opportunity for savings. Indeed, initiatives such as REMEDY at Duke University recover unused supplies and distribute them to developing countries.

The research literature on materials management in hospitals is limited. Moreover, the application of traditional manufacturing-based analytical models for materials planning in hospitals is often insufficient and leaves crucial issues unaddressed. Consequently, hospital materials managers often make decisions by gut feeling rather than by a structured decision process. High material availability rates (and high material costs) are achieved through stocking excessive quantities and by frequent expediting.

**Significant opportunities exist to reduce waste by improving the planning and coordination of materials and information in hospital operating theaters.**

In this article, we develop a model and solution approach to help reduce hospitals’ inventory investments and materials management costs while maintaining high availability rates in surgical operating rooms, known as the operating theater (OT). We believe there are opportunities to improve inventory performance by analytically planning and coordinating materials with surgical schedules and by formalizing the materials management decision process.

Although our research addresses only part of the reason for rising health care costs, reduced materials management costs and better inventory performance will allow hospitals to invest cash in new technologies, procedures, and improvements in patient care instead of maintaining excess inventory in a stockroom. This issue is therefore of strategic interest to the senior leadership team.

We have two research objectives. First, we want to extend the existing body of inventory research through novel modeling and solution approaches to problem attributes found in the hospitals we examined. Our model complements existing research and models for optimizing patient, physician, and operating room planning and scheduling. Second, we want to create a general modeling framework with a strong analytic foundation that considers uncertainty explicitly and that can be used to optimize materials preparation, placement, and management costs.

The framework is designed to be part of a process to answer tactical and operational questions, quantify trade-offs between alternatives, evaluate the impact of system changes on material requirements, and explore potential materials management initiatives. We observe that many materials managers do not know how to approach tactical questions such as these: How much should the hospital expect to invest in inventory? What materials should the hospital invest in to provide the best service given a fixed budget? What materials should be purchased and stocked and which should be obtained via vendor consignment? What is the value of incorporating the nonstationary and stochastic behavior of demand for materials into planning hospital inventory levels?

Our framework also can be used to explore improvement initiatives that bridge information gaps, scheduling policies, and materials policies by answering the question: What savings are provided by standardizing the materials used by all physicians for a given procedure? From our interviews, surgical department physicians and
managers are very interested in knowing the potential value of standardizing procedures and materials for a small number of surgeons.

Although some surgeons value the freedom to choose material preferences, there is no formal way of evaluating the costs of these preferences. Our model can quantify the financial consequences for individual surgeons. To be effective, it is imperative that our modeling framework be flexible, scalable, and fast. Flexibility is necessary to factor in additional considerations. Scalability is required due to the large number of items, time periods, and resource combinations. Speed is essential to provide a management team with rapid what-if exploratory decision possibilities.

The remainder of this article is organized as follows. First, we review the relevant literature. Then, we discuss the complexities and unique challenges of materials planning in several hospitals. We formulate our model and present a numerical study and managerial insights. Finally, we conclude with a discussion of future research directions.

**Literature Review**

Past research pertaining to hospital inventory management can be grouped into three general categories: (1) economic and industry discussions regarding growth in health care costs, pressure to lower hospital operating expenses, and the need for improved approaches and analytical techniques; (2) analytical models based on reorder point (ROP)/economic order quantity that are used to set steady-state reorder points/lot sizes and materials resource planning (MRP) systems that assume deterministic demand and processing lead times; and (3) case studies and empirical evidence of initiatives and policies used to estimate cost savings. We review and discuss our research contribution to the existing literature.

Discussions and longitudinal studies that motivate the need for improved approaches to manage hospital inventories are found in Jarett (1998), Roth and Van Dierdonck (1995), and Aptel and Pourjalali (2001). Harper (2002) provides a useful discussion and process-based framework for improving the organizational structure for the use of hospital resources. Our work builds on these discussions and is unique in that we formulate a quantitative model that may be used to quantify the benefits of such organizational changes.

The application of reorder point/order quantity (ROP/OQ) inventory models in hospitals is discussed in Taylor (1990) and Prashant (1991). These are commonly referred to as “par level systems” in hospitals and are widely applied in practice due to their simplicity of implementation. However, we observed that the use of par levels and the setting of ROP/OQ values are often made by a planner’s personal judgment and experience rather than through a structured decision process based on data. Veral and Rosen (2001) discuss this observation and how the experience-based approach of managing inventory often leads to overstocking and excessive expediting of materials. Hassan et al. (2005) and Burns and Tucker (2001) develop hospital inventory replenishment models based on assuming deterministic demand. Dellaert and Van De Poel (1996) and Nicholson et al. (2004) propose models with stochastic demand and service levels that address the critical nature of stock-outs in health care settings. Nicholson et al. (2004) model a single-item, single-period, multi-echelon formulation to set par levels of noncritical items in a hospital. The model is an arborescent structure, in which inventory is not shared across departments. The introduction of service-level constraints in the multistage environment makes the problem NP-hard due to a nonlinear objective function with a nonconvex set of constraints. A greedy heuristic is proposed and exploits the newsvendor structure of the problem. Dellaert and Van De Poel (1996) consider demand with Poisson arrivals and normally distributed transaction sizes and propose a multi-item, single-echelon, single-period model. The performance of this model is compared to a two-step method based on the Markov chain approach of Ferdergruen et al. (1984).

The application of MRP systems to hospital inventory control is discussed in van Merode et al. (2004), Rossi-Turck et al. (2004), and Showalter (1987). MRP is applied to hospitals through diagnostic-related groups (DRGs). A DRG is a patient classification system based on similar materials and resource requirements. Independent variables for sorting the DRGs include surgical procedures or the combination of surgical procedures and ranges of patient ages. Patients within a DRG are clinically similar and treatment of patients within this classification requires the same bill of materials (BOM). However, the implementation of MRP logic in hospitals has shortcomings when dealing with issues such as demand and BOM uncertainty due to patient or diagnosis changes. Moreover, routings are difficult to specify. Roth and Van Dierdonck (1995) provide a comprehensive discussion of MRP challenges and develop a hospital-based planning and control system called hospital resource planning (HRP).

To our knowledge, Roth and Van Dierdonck (1995) are also the first to discuss the stochastic nature of the BOM during surgical treatments. The stochastic bill of materials (SBOM) differs significantly from traditional manufacturing logic and highlights a challenge of applying MRP logic directly to hospital settings. Our research is the first to extend this discussion and incorporate an SBOM into a rolling-horizon planning model. We can quantify the impact of SBOM on inventory requirements and potential financial savings of reducing the stochastic nature of the BOM through standardization.
Research examining strategic inventory initiatives in hospitals includes Jarett (1998), Rivard-Royer et al. (2002), and Epstein and Dexter (2000). Jarett (1998) discusses the application of just-in-time (JIT) principles to hospitals. Rivard-Royer et al. (2002) note that the benefits of using JIT principles include a reduced investment in high-cost items, improved communication with suppliers, and a focus on product standardization for procedures. Rivard-Royer et al. (2002) document these savings in the form of a case study, and Epstein and Dexter (2000) discuss opportunities associated with linking the operating room (OR) scheduling information system to the materials management system to apply JIT inventory management.

Our research also focuses on improved information sharing and reduced inventory investment; however, we do not endorse a JIT philosophy for all items due to the highly uncertain, nonstationary nature of demand. Our model uses an OT schedule of patients, physicians, operating rooms, and ancillary resources as input. Past research in the optimal scheduling of surgical procedures has been structured to maximize resource utilization (operating rooms, beds, physicians) subject to a set of constraints. Belien and Demeulemeester (2007) present a formal literature review of different aspects of hospital scheduling problems and Sier et al. (1997) provide an excellent discussion of the factors and challenges that practitioners face when developing a surgical schedule. Blake and Donald (2002) present a case study of a specific hospital, its scheduling challenges, and a quantitative approach to improve its scheduling and operational performance. Other examples of approaches to solve the surgical scheduling problem are discussed in Jebali et al. (2006), Chaabane (2004), and Fei (2005).

Scheduling models that incorporate stochastic attributes include Kolesar (1970) and Gerchak et al. (1996), who present a stochastic dynamic programming model and determine the elective surgeries that may be performed each day depending on the required procedures of that day. Belien and Demeulemeester (2007) consider both a stochastic number of patients and a stochastic length of stay and use a mixed-integer programming approach to plan daily surgeries and bed occupancy. Our research extends the scheduling literature by determining an optimal material support plan. In this environment, optimal inventory levels are dynamic and depend on the OT schedule, the SBOM, available preparation capacity, economic costs, and procedure schedule volatility. Much of the past research in inventory theory is concerned with the optimality of control policies. We are not interested in the form of the optimal control policy that a dynamic programming approach would yield. From the inventory theory literature, our work is closest to the problems addressed in Kapuscinski and Tayur (1998) and Sox and Muckstadt (1997). Metters et al. (2006) examine the form of the optimal policy for stochastic nonstationary demand. Kapuscinski and Tayur (1998) approach the problem using infinitesimal perturbation analysis.

In contrast, we are interested in computing optimal decisions quickly in the presence of uncertainty. Our approach is suitable for large-scale systems with thousands of items and dozens of time periods. Furthermore, because we employ linear programming methods, it is straightforward to customize the model to fit a specific hospital’s need by adding constraints.

Our research makes three contributions to the existing literature. First, we develop a general multi-echelon, multi-item, capacity-constrained materials planning model that considers nonstationary, highly uncertain demand. Second, the model considers SBOMs and can be used for tactical and operational decisions on a rolling-horizon basis. Third, because of the problem formulation, our model provides integer solutions and solves in a few seconds on a personal computer.

**Problem Description**

We will focus on the materials management in the collection of surgical operating rooms known as the operating theater. The OT stocks inventory that is often the most critical to manage in terms of cost and availability. The OT is supported by a central inventory location that is nearby or within the OT, known as the “core.” The core is often physically constrained by its storage space, so it is resupplied periodically (typically daily) from a larger on-site central processing (CP) and storage location. The core supports the operating rooms with the inventory needed for each procedure. CP contains a wide variety of material and serves as a preparation area to organize, sterilize, and build procedure-specific kits. These kits are groups of items common to an individual surgeon’s procedure. The specific items contained in a kit are rarely standardized and are specified by individual surgeons. CP carries material in either unprepared or prepared form. Finally, CP is resupplied from a supply base of various vendors. The material flow system between each echelon, or level, in the system under study is shown in Figure 1.

Materials planning in the OT requires high availability for expensive items in spatially constrained locations and in the presence of nonstationary, uncertain demand. The materials vary by cost, annual volume, short-term demand uncertainty, and perishability. Demand uncertainty arises, in part, because the materials required for a patient procedure are a function of both the patient and the surgeon. The unique requirements of the patient, the surgeon’s preferences for materials, and the actual consumption of those materials by the surgeon during the treatment can differ dramatically for each patient and each surgeon. In
addition, the scheduling of surgeons for their services often generates large peaks and valleys in the demand processes for the materials. Insufficient material availability can result in delays that may create several undesirable consequences: (1) additional labor costs associated with preparing, transporting, and handling emergency replenishments from CP or external sources; (2) higher expedited freight costs from external suppliers; (3) postponing and rescheduling patient treatment and physician capacity to a future time; or (4) significant change in the patient’s condition. We consolidate all of these costs into a “backorder” cost. As a result, a practical reality is that the OT attempts to achieve a very high availability rate by stocking enormous amounts of inventory, incurring high carrying costs, and exposing the system to financial risk; however, space, labor, and financial resources are constrained. We briefly highlight the function of each echelon of material flow, the purpose of the inventories held, and the challenges that each echelon must address. These are summarized in Table 1. We follow with a discussion of these issues in greater detail. In sections 3.1 and 3.2, we discuss the nature of demand. In section 3.3, we describe some attributes of the environment and the organizational disconnects we observed in hospitals.

**Stochastic Bill of Materials**

Similar to Roth and Van Dierdonck (1995), we observe a unique analytical challenge that is common to hospital settings: an SBOM for surgical procedures. As we demonstrate, this issue has a more devastating impact on inventory requirements than is intuitively recognized. The SBOM for a procedure is caused by a physician-determined “preference card” or “preference list.” It records the physician’s preference for materials and the patient’s need for materials to support the procedure. However, depending on how the patient procedure evolves, the materials on the SBOM may be used in varying quantities or perhaps not at all.

Departments that contain dozens of physicians with different types of medical training, techniques, levels of experience, and technological preferences can generate thousands of unique preference cards with varying rates of consumption. For example, one hospital in our study has over 8,000 preference cards on file. Over half of the cards have not been actively used in the past two years because either physicians and techniques have changed or the procedure and requirements are rare. However, because the preference card exists in the system, the system holds some quantity of several infrequently used items. How should a materials manager determine safety stock requirements for an item when it is not known how much a department will use for a given procedure or whether the item will be needed at all? What safety stock parameters should be entered into the hospital’s planning system?
The SBOM and the stochastic nature of the patient procedure make it exceptionally difficult for materials managers to plan inventories and properly meet service requirements given physically constrained stocking locations and financially constrained inventory budgets. We observe that materials managers qualitatively understand the importance of reducing the number and complexity of SBOMs, yet they struggle to quantify the financial impact of SBOMs, surgical preference card proliferation, and the benefits of standardized preference cards for a surgical department. A major contribution of our work is that we explicitly model the SBOM and its effect on inventory levels as part of a decision framework. Our model may be used by management to quantify the financial impact of the lack of standardized preference cards.

Block Scheduling and Nonstationary Demand

The materials management challenges that arise in hospitals are further complicated by the stochastic and nonstationary nature of individual item demands. Although most surgical procedures are scheduled weeks in advance, a patient’s condition and material requirements may change at any point after diagnosis or during the procedure itself. Furthermore, materials planners cannot predict daily emergency surgical procedures accurately. Therefore, there are two sources of demand uncertainty: the stochastic demand for daily procedures and the SBOM created by a physician’s lack of standards. These two sources interact to create very high demand uncertainty at an individual item level. Although some of the materials planners we interviewed attempt to apply traditional steady-state inventory models for managing inventory, such as reorder point/order quantity policies, actual inventory levels exceed these levels due to manual overrides. The consequences are predictable: a CP and OT core that are filled with inventories to the point of requiring the virtually constant nonvalue-adding activities of counting, sorting, and checking.

In addition to high uncertainty, demand for surgical materials in an OT exhibits a nonstationary behavior. The finite number of operating rooms, physicians, nurses, and staff at the hospitals we observed are planned with block schedules by the surgical department. For example, an orthopedics department that we interviewed schedules several procedures for its physicians only on specific days of the week. Blake and Donald (2002) provide a detailed example of a surgical block schedule at a major hospital in Canada.

While block scheduling is convenient for labor planning and improves the utilization of key resources, the approach generates nonstationary demand surges for materials that are often ignored and treated as “uncertain.” This is particularly costly for high-cost items. When combined with high demand uncertainty and traditional inventory management policies, these surges degrade the performance expectations of steady-state inventory policies. High-cost orthopedic materials may be in high demand on one day every two weeks and in very low demand for the rest of that time. However, the high service requirements of the item coupled with its unpredictable demand will drive a large amount of safety stock for that item. We observed this discrepancy in all of the hospitals we studied.

Past research on nonstationary stochastic demand has focused on structural properties of the optimal control policy and not on providing fast, optimal solutions to large-scale practical problems.

Schedule Coordination and Information Accuracy

While traditional reorder point inventory policies are simple to implement, they replenish past demand and do not inherently consider planned future demand. Therefore, not only are hospital materials managers contending with highly uncertain, nonstationary demand, but the inventory policies being used do not explicitly consider the planned demand attainable from the block schedule.

The block schedule format provides potential advantages to materials managers. Although changes in the schedule and procedure can and do occur, a high percentage of the materials requirements are readily known weeks in advance from the block schedule. Unfortunately, a disconnection exists between

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**Figure 2**

(a) Observed and (b) Proposed Flow of Information.

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**Table:**

<table>
<thead>
<tr>
<th>Functional Group</th>
<th>Purchasing</th>
<th>Preparation/Reprocessing</th>
<th>Scheduling</th>
<th>OR Staff</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Observed Information Flow</td>
<td>Shipments due</td>
<td>Procedure list (BOM)</td>
<td>Next period schedule</td>
<td>Procedure &amp; Patient Data</td>
</tr>
<tr>
<td></td>
<td>Inventory Data</td>
<td>Inventory levels</td>
<td></td>
<td>Charged consumption data</td>
</tr>
<tr>
<td>(b) Proposed Information Flow</td>
<td>Shipments due</td>
<td>Procedure list (BOM)</td>
<td>Next period schedule</td>
<td>Procedure &amp; Patient Data</td>
</tr>
<tr>
<td></td>
<td>Inventory Data</td>
<td>Inventory levels</td>
<td></td>
<td>Actual consumption data</td>
</tr>
</tbody>
</table>
scheduling surgical departments and the materials management organizations at the hospitals we examined. Although advanced scheduling information exists, materials managers are unaware of the specific procedures and the physician’s preference card requests until the day before the procedure.

Figure 2(a) diagrams organizational roles we observed within hospitals and specific types of information sharing between functional groups. Typically, only the next day’s schedule is shared with the personnel who prepare and process the preference card for each procedure. This leads to excessive safety stock and reliance on expediting materials when advanced demand information is readily knowable. Although schedules are adjusted daily, every hospital in our study stated that a large majority of the procedures on a given day were known several weeks in advance. Figure 2(b) proposes a higher level of information sharing between the scheduling department and materials planners. In addition to the next day’s schedule, the scheduling department can provide future scheduled procedures with the materials planners. Our framework can quantify the value of increased information sharing on inventory levels and expediting, as detailed below.

Another challenge in hospital materials management involves inventory tracking and the accuracy of material consumption. Excess materials often are not accurately tracked in inventory planning systems, nor are they returned to the proper stocking location. This issue was described during an interview with the head OT nurse at a hospital, who bluntly stated, “Doctors and nurses are not stock clerks.” In addition to the unpredictable consumption of materials, there is a lack of discipline and perceived need to track actual material consumption properly. This creates both an information gap and a gradual imbalance of inventory in the space-constrained OT core. This situation can be controlled only with a labor-intensive (and therefore expensive) process of cycle counters visually tracking reorder point levels and expediting needed materials.

While our modeling framework can be applied to quantify the value of sharing information between departments, hospital managers must improve the discipline of tracking actual material consumption to reduce their reliance on costly cycle counting and expediting.

**The Model**

The model takes as input the most up-to-date block schedule and SBOM and provides optimal material purchases from external suppliers, central processing capacity allocation, and inventory placement at all echelons. Three possible forms of inventory may be carried in the system: unprepared, prepared, and OT core inventory. A representation of the flow of materials over time is shown in Figure 3. Conservation of inventory flow is assumed in which beginning inventory levels plus received inventory minus inventory consumed or transferred equals the ending inventory level. The periodic sequence of events is as follows:

1. Materials are received at the CP and OT.
2. Demand is realized and materials are consumed at the OT.
3. Inventory holding costs and shortage (expediting) costs are assessed.

### Figure 3

Materials Flow Interdependency from the Suppliers to CP and OT.
4. System information about the OR schedule and OT inventory (in the event that expedited material was added or expired material was discarded) is updated and a new materials plan is computed.

5. The next period’s replenishment order is confirmed for CP preparation and delivery at the beginning of the next period (subject to material and capacity availability).

6. Replenishment purchase orders are placed with external suppliers for delivery after some known and constant lead time.

7. The inventory at the end of the period is carried forward.

Preliminaries

Let \( |a|' = \max \{a, 0\} \) for a real and let \( Z \) denote the set of non-negative integers.

Indices and sets

\[ t \in T = \{1, 2, \ldots, |T|\} \text{ index of the time periods in the planning horizon, where } t = 1 \text{ refers to the current time period}; \]
\[ j \in M = \{1, 2, \ldots, |M|\} \text{ index of the block scheduled OR procedure types}; \]
\[ i \in P = \{1, 2, \ldots, |P|\} \text{ index of the items used in the delivery of the OR procedures}; \]

Item data

\( l \), the known and constant supplier lead time for item \( i \) to the CP; \( h_i^t \) the installation holding cost for the item \( i \) per unit per period held in unprepared form; \( h_i' \) the installation holding cost for item \( i \) per unit per period held in prepared form at the CP, where \( h_i > h_i^t > 0 \); \( h_i \) the installation holding cost for item \( i \) at the OT, where \( h_i > h_i^t \); \( b \) the cost per unit per period of an emergency replenishment to the OT;

Block Schedule, Material Demand, and Inventory Data

\( C \) the total available CP preparation capacity in period \( t \); \( d_p^t \) (block schedule) discrete, non-negative random variables (r.v.) representing the planned and forecast number of treatments of type \( j \) scheduled in period \( t \); \( u_i \) (stochastic bill of materials) discrete, non-negative r.v. representing the quantity of item \( i \) used in a single treatment of type \( j \); \( d_{it} = \sum_i u_{ij} d_j^t \) the r.v. representing the quantity of item \( i \) required to perform all treatments in period \( t \), with mean \( \mu_i \) and variance \( \sigma_i^2 \); \( d_{it} = \sum_{n=1}^{\infty} d_{in}^t \) the cumulative demand for item \( i \) through \( t \); with mean \( n \) and variance \( \sigma_I^2 \); \( r_{it} \) the scheduled receipts from the supplier of item \( i \) in \( t = 2, \ldots, l_i \) associated with previous replenishment orders placed in periods \( t - l_i \); \( AR_{it} \) the initial available on-hand inventory of item \( i \) in unprepared form at the CP; \( AQ_{it} \) the initial available on-hand inventory of item \( i \) in prepared form at the CP; \( AS_{it} \) the initial net inventory level of item \( i \) at the OT;

Material Planning Decisions

\( r_t \) the replenishment order quantity decision from the supplier to the CP for item \( i \) made in \( t - l_i \) for delivery in \( t = l_i + 1, \ldots, T + l_i + 1 \); \( q_t \) the CP preparation quantity decision of item \( i \) in period \( t \); \( s_t \) the CP to OT transfer quantity decision of item \( i \) made in \( t \) for delivery in \( t + 1 \); \( R_{it} = \sum_{n=1}^{\infty} (R_{in} + r_{it}) \) the cumulative receipts of item \( i \) from the supplier through \( t \); \( Q_{it} = \sum_{n=1}^{\infty} q_{in} \) the cumulative quantity prepared of item \( i \) in the CP through \( t \); \( S_{it} = \sum_{n=1}^{\infty} s_{in} \) the cumulative quantity of item \( i \) transferred to the OT through \( t \); \( CR_{it} = AR_{it} + R_{it} \) the cumulative supply of unprepared receipts of item \( i \) through \( t \); \( CQ_{it} = AQ_{it} + Q_{it} \) the cumulative supply of prepared of item \( i \) in the CP through \( t \); \( CS_{it} = AS_{it} + S_{it} \) the cumulative supply of item \( i \) transferred to the OT through \( t \);

We assume that the random variables \( u_i \) are independent across material type \( i \) and treatment type \( j \). We assume that \( u_{ij} \) and \( d_{ij} \) are independent. Thus, the convolved demand for items \( d_i \) is independent across time periods and items. This is an approximation because events in the OR can cause collections of materials to be used in excess, thus creating positively correlated demand. Such positive correlation reduces the risks of inventory imbalances over the planning horizon, as shown in Erk et al. (1990). However, some materials are stocked for contingencies in case unplanned circumstances arise, thus creating negatively correlated demand. We assume independence for model tractability and because we believe that it will not significantly affect the quality of our planning decisions.

We assume that the planning horizon \( T \geq \max\{|l_i|\} \) is the longest lead time for an item. Note that the decisions encompass both firm and planned material replenishment orders. For planning purposes, unsatisfied demand is assumed to be fully backordered.

We consider the backorder cost to be the excess material handling cost associated with having to requisition additional material expeditiously. This assumption does not pose any detrimental practical issues because we assume that, at the end of each period, system information is updated with the most up-to-date inventory levels and a new plan is computed. For items with extremely long lead times, this may present an inventory imbalance situation in which there is more inventory on order than is desired; however, for these items, the coefficient of variation of the demand during the lead time decreases rapidly, thus lowering the likelihood of such an event.

Because some items have very low demand rates and require integer decisions, we model item-level demand by a random variable \( d_i \) with probability mass function \( P_i \{d_i = x\} \times Z \). All moments of \( d_i \) are assumed to exist and be finite. Its first two moments are

\[ \mu_i = E(d_i) = \sum_j E(d_{ij})E(\mu_{ij}) \]
\[ \sigma_i^2 = \text{Var}(d_{ij}) = \sum_j \left( \text{Var}(d_{ij}).E(u_{ij}) \right)^2 + \text{Var}(u_{ij}).E(d_{ij})^2 + \text{Var}(d_{ij}).\text{Var}(u_{ij}) \]

Equation (4.2) is the variance of the sum of the product of independent random variables and will be used to analyze the effect of SBOMs. Depending on whether the item’s variance-to-mean ratio (VTMR) is greater than one, equal to one, or less than one, we approximate the demand random variable using a negative binomial, Poisson, or binomial probability model, respectively.

Without a loss of generality, we assume that the length of a time period has been defined in accordance with the frequency of material deliveries from CP to the OT and that the material demands and supplier replenishment lead times are in multiples of this base time period. It is assumed that we are optimizing the inventory level for expensive items for which the economic lot size for handling is one unit. Thus, we assume that there are no set-up times that affect lot-sizing decisions each period.

An Unconstrained Single-Echelon Expected Cost Function

We develop a single-echelon optimization model that minimizes the expected costs at the OT based on the assumption of unconstrained (infinite) supply from CP. We will use this single-echelon model as a building block in the formulation of the entire model. The single-echelon model will consider uncertain and nonstationary nature of the demand process, there may be more inventory left over at the end of a period than is desired in the next period. This is because it is assumed that inventories may be reduced only through demand. Minimizing (4.3) takes the interdependence of decisions between time periods through the use of cumulative supply variables and demand random variables. This modeling technique was used by Sox and Muckstadt (1997). However, (4.3) ignores the interdependence of decisions among the OT, CP, and the supplier for each item. Furthermore, due to auxiliary resource capacity limitations (such as labor), there may be periods in which the OT is unable to perform certain treatments. In these cases, it is optimal to postpone transferring materials to the OT. Finally, a transfer capacity limitation may exist in which only a maximum number of units can be transferred to the OT per period. With multiple capacity constraints, there may be periods during which inventory levels cannot be raised to desired levels. These issues are referred to as the “trough-peak” and the “peak-trough” problems in Metters, R. et al. (2006). Material availability at the OT is dependent on material availability at CP. It may not always be possible to supply the desired quantities from CP’s stock. Our decision model formulation must be flexible enough to handle each of these issues.

Note that solving (4.3) is fast via marginal analysis because of its convexity. Let \( CS_i \) denote its optimal solution. For integer \( x \), we define the forward-first difference function of \( G_i(x) \) as

\[ \Delta G_i(x) = G_i(x + 1) - G_i(x) = (b_i + h_i) \text{Pr}[D_t \leq x] - b_i \]

Note that \( G_i(x + 1) \geq \Delta G_i(x) \), for integer \( x \). For computational efficiency, we may use a piecewise linear representation of (4.5) reformulate (4.3) as

\[ \min_{x_{itk}} \sum_{i \in P} \sum_{t \in T} G_i(C_{St}) \]

s.t. \( C_{St} \geq C_{St-1} \geq 0 \) for \( t = 1, \ldots, T \)

where

\[ G_i(x) = (b_i + h_i) \sum_{i=0} \left( x - k \right) \text{Pr}[D_t = k] + h_i(\bar{u}_i - x) \]

is the familiar convex newsvendor cost function.

Given the nonstationary nature of the demand process, there may be more inventory left over at the end of a period than is desired in the next period. This is because it is assumed that inventories may be reduced only through demand. Minimizing (4.3) takes the interdependence of decisions between time periods through the use of cumulative supply variables and demand random variables. This modeling technique was used by Sox and Muckstadt (1997). However, (4.3) ignores the interdependence of decisions among the OT, CP, and the supplier for each item. Furthermore, due to auxiliary resource capacity limitations (such as labor), there may be periods in which the OT is unable to perform certain treatments. In these cases, it is optimal to postpone transferring materials to the OT. Finally, a transfer capacity limitation may exist in which only a maximum number of units can be transferred to the OT per period. With multiple capacity constraints, there may be periods during which inventory levels cannot be raised to desired levels. These issues are referred to as the “trough-peak” and the “peak-trough” problems in Metters, R. et al. (2006). Material availability at the OT is dependent on material availability at CP. It may not always be possible to supply the desired quantities from CP’s stock. Our decision model formulation must be flexible enough to handle each of these issues.

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Note that \( G_i(x + 1) \geq \Delta G_i(x) \), for integer \( x \). For computational efficiency, we may use a piecewise linear representation of (4.5) reformulate (4.3) as

\[ \min_{x_{itk}} \sum_{i \in P} \sum_{t \in T} \left( G_i(0) + \sum_{k=0}^{k_0} \Delta G_i(k).x_{itk} \right) \]

s.t. \( 0 \leq x_{itk} \leq 1 \),

where

\[ x_{itk} = \begin{cases} 1, & \text{if } \Delta G_i(k) < 0, \\ 0, & \text{otherwise} \end{cases} \]

and the optimal stock levels for each item \( i \) in \( t \) are the newsvendor solutions given by

\[ CS_i = \sum_{k=0}^{k_0} x_{itk} \]

Note that constraint (4.4) does not appear explicitly in (4.7) because it is a consequence of the structure of the marginal costs, as shown in Lemma 1. Observe that \( G_i(x) \) measures the expected incurred cost for item \( i \) in period \( t \) based on the expected difference between cumulative supply and demand random variables. Considering the dependence of stocking decisions between periods, it may be desirable to have the inventory levels be higher or lower than a single-period optimal newsvendor solution in order to minimize the total costs during the planning horizon. The optimal decisions will balance current expected backorder costs with future period expected holding costs, resulting in an inventory smoothing effect.

**Lemma 1:** \( \Delta G_{it-1}(k) \leq \Delta G_i(k) \leq CS_i \geq \Delta G_i(0) \)

**Proof:** For ease of exposition, the subscript \( i \) is suppressed. We know that \( \Delta G_{it-1}(k) \leq \Delta G_i(k) \) because the demand random variables are non-negative and consequently for any \( k, P, \{D_t \leq k\} \leq P, \{D \leq k\} \), fort = 1, 2, ..., T - 1. From (4.9), let \( k_0 = \inf \{k: \Delta G_i(k) \geq 0\} \) and \( k_1 = \inf \{k: \Delta G_{it-1}(k) \geq 0\} \). Note that \( k_0 = CS_i \) and \( k_1 = CS_{i-1} \). Since \( \Delta G_{it-1}(k) \leq \Delta G_i(k) \leq k_1 = k_0 \)
Because the $CS_t$ are cumulative stock levels as $s_0 = CS_0 - CS_{s-1}, s_t = 1,2, \ldots, T$. The problem formulation takes into consideration the risks associated with excess inventory remaining after a peak period. However, this formulation assumes unlimited material availability from CP. The advantage of this formulation is its extremely simple structure and computational simplicity.

The Complete Model Formulation

Next, we extend the base model to a multi-echelon setting. We incorporate material and capacity availability at CP and the replenishment order from the supplier for delivery $l_i$ periods in the future for item $i$. We assume that the replenishment lead time is fixed and known. Let $AR_{i0}$ and $AQ_{i0}$ be the beginning on-hand inventories in CP of item $i$ in prepared and unprepared forms, respectively, after the current peak period. However, this formulation assumes unlimited material availability from CP. The advantage of this formulation is its extremely simple structure and computational simplicity.

Numerical Study and Managerial Insights

The accuracy and quality of the decisions resulting from implementing rolling-horizon planning models such as (4.11) have been demonstrated in past research such as Rappold and Yoho (2006) and Caggiano et al. (2006) via simulation techniques. Leveraging these results, the purpose of this numerical experiment is to explore the potential savings in OT inventory and material management costs resulting from structural changes to the system, such as the standardization of the BOM. Our basis of comparison will be expected holding and backorder costs.

Design of Numerical Study

Using representative data of hospitals’ operating theaters, we consider an environment of 26 materials used in various quantities in 81 patient procedures that are divided among four surgical departments. The cost and usage data used were from a combination of the implant manufacturer (for cost data) and from a hospital (for usage and SBOM data). The procedures in the surgical departments are scheduled in a block schedule format during a cycle of live time periods (work days). The average cost of an item is US $206 and ranges from US $31 to US $2,531. These were the manufacturer’s unit costs and the manufacturer owns the inventory until it is consumed. The hospital’s standard costs for the same items are considerably higher but do not reflect accurately the cost of carrying the inventory. Therefore, we chose to use the manufacturer’s unit cost for valuing the inventory. The total average demand for the items is 37.2 units per period and varies between 29.9 and 47.2 across the work week. For each scenario, we consider a time horizon of 90 period $t$.

We consider four factors in the design of experiments (DOE): (1) the stochastic OR block schedule, (2) the stochastic bill of materials, (3) the average CP preparation capacity utilization, and (4) the backorder-to-holding cost ratio of materials in the OT core. The stochastic schedule and BOM are modeled by varying their VTMR, with $VTMR = 0$ representing certainty. The OR schedule VTMR is used to model differences in OR procedures in the surgical departments and materials management increases, the OR schedule VTMR decreases. The BOM VTMR is used to model improved BOM standardization among the physicians. The factors and values are listed in Table 2.

Table 2

<table>
<thead>
<tr>
<th>DOE Factor</th>
<th>Range of Values</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>OR Schedule VTMR</td>
<td>0</td>
<td>1 5 10</td>
</tr>
<tr>
<td>BOM VTMR</td>
<td>0</td>
<td>1 5 10</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>75%</td>
<td>85% 95%</td>
</tr>
<tr>
<td>BO-HC Ratio</td>
<td>9.1</td>
<td>99.1</td>
</tr>
</tbody>
</table>

Actual holding cost rates were disputed and unavailable from the hospitals we interviewed. For comparison purposes, we estimate the holding cost rates for the unprepared, prepared, and transferred material. (4.13) considers capacity limits and decision linkages among unprepared, prepared, and transferred material.

This computationally efficient modeling framework incorporates nonstationary uncertain demand; solves quickly using off-the-shelf algebraic modeling software, such as Dash Optimization’s Xpress; and allows additional constraints to be added, such as space limitations.
transferred echelon as 15%, 20%, and 25%, respectively. With such small echelon holding costs, we would expect most safety stock to be positioned forward in the OT core; however, in a real application, space constraints in the OT core may prevent this.

Numerical Study Results and Managerial Insights

Our results are summarized in Tables 3 and 4. For both backorder-to-holding cost (BO:HC) levels, a decreased OR schedule, VTMR, and BOM VTMR play a significant role in reducing the total expected cost of the system; however, the relative change in expected costs did not depend significantly on the particular BO:HC ratio.

Interestingly, when the BOM VTMR is zero, OR schedule VTMR increases expected costs by less than 9.1%. However, when the OR schedule VTMR is zero, the BOM VTMR increases expected costs by 36.5% for the BO:HC = 9:1 ratio case. The presence of both sources of uncertainty interact to increase expected costs by 89.5% for the BO:HC = 9:1 ratio case. This illustrates the potential economic impact of the SBOM. Each situation will differ, depending on the structure of the SBOM; however, the increase in demand uncertainty due to (4.2) and the associated inventory costs are significant.

A possible way to reduce the BOM VTMR is by incorporating DRG classifications, as discussed in Roth and Van Dierdonck (1995), into the scheduling of procedures and therefore into our model. When patients are scheduled for specific procedures several weeks in advance, the DRGs, based on attributes such as patient age, can be incorporated as a way to reduce the variance of the BOM required for the procedure.

The cost effects of limited capacity in our study were dominated by the OR schedule, VTMR, and BOM VTMR. Table 4 shows the relative impact of increased capacity utilization on the expected system costs. Increased capacity utilization does increase the absolute costs in two ways. First, if the materials preparation stage is highly utilized on average, then the expected number of backorders will increase. Second, to offset the likelihood of capacity-induced material shortages, more inventories are held in the prepared state and at the OT. Capacity constraints will affect the ability of CP to supply materials to the OT and the positioning of the inventory.

Figure 4 is an example of how materials policy decisions affect planned inventory levels during several time periods. The straight dashed line represents a reorder point (ROP) inventory policy used by the hospitals we interviewed. Our rolling-horizon policy (RHP) is represented by the solid line. By accounting for the nonstationarity of the scheduling of procedures, we are able to proactively manage the uncertainty and position safety stock dynamically. Finally, a more advanced RHP is shown by the dashed line. In this case, variance reduction initiatives (through BOM standardization and information sharing) further reduces the need for holding inventory.

Although information sharing between the surgical department and materials management staff...
offers benefit, standardizing the BOM and reducing the uncertainty imbedded in the preference card philosophy offers the most significant opportunity for inventory carrying and materials management cost savings.

Model Flexibility and Extensions
From our observations of various hospital settings, we believe our modeling framework is flexible enough to accommodate the peculiarities of individual hospitals. The available preparation capacity in CP is usually constrained by labor. While we defined Ct as an input, we may also make it a decision variable to help with short-term labor-planning decisions. Letting wt be the variable cost per labor unit (e.g., labor hours), we add the term wtCt to the objective function. Similarly, space (or volume) and financial budget limitations can be incorporated by first defining an item’s per-unit volume and unit cost as vi and ci, respectively, and then adding the corresponding constraints.

Another model application addresses outsourcing hospital materials replenishment systems. In this case, a third party such as a distributor kits all items for scheduled procedures and delivers the procedure-specific kits directly to the hospitals. Rivard-Royer et al. (2002) also discuss how a hybrid approach that incorporates stockless policies may produce marginal savings. We observe that companies still offer this type of service to hospitals. Our approach can be used to evaluate such an approach and determine for which items this makes the most economic sense. The original objective function then includes the unit costs of the items both prepared by the distributors (at a markup) and the cost if purchased and prepared at the hospital. In addition, the cost of labor associated with the preparation process is included in the objective function to properly consider a possible reduction in labor requirements. There is an inherent trade-off between the unit cost of the item and the variability associated with stocking and holding the item.

Summary
This article examines materials planning issues that arise in hospital operating theaters. We developed a rolling-horizon model to optimize operating theater inventories while respecting high availability rates and resource constraints. We model demand for surgical procedures as nonstationary and we incorporate a stochastic bill of materials for each procedure. We conducted a numerical study and sensitivity analysis to pinpoint the leverage points in the system for reducing inventory carrying and materials management costs. Reducing the uncertainty in the BOM exhibited the greatest potential for savings. By leveraging knowledge of future scheduling information and having a disciplined approach to monitor the consumption and allocation of inventory, it is possible for hospitals to reduce their investment in inventory, labor dedicated to cycle counting, and over-reliance on expediting. Although hospitals currently may not have this level of coordination, our modeling framework may be leveraged to determine the value of coordinating interorganizational functions such as OR scheduling into materials management and reducing the uncertainty of the bill of materials.

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About the authors

**Dr. James A. Rappold** is President and CEO of Supply Chain Sciences, a strategic management consultancy and provider of advanced decision support software. Prior to Supply Chain Sciences, Dr. Rappold spent 12 years as a professor and director of the Supply Chain Executive Education programs at the University of Wisconsin-Madison. Dr. Rappold holds a B.S. in Industrial Management and Mathematics from Carnegie Mellon University and an MS and Ph.D. in Operations Research and Industrial Engineering from Cornell University.

**Dr. Ben Van Roo** is a Senior Scientist at SignalDemand, a San Francisco, CA based software firm that focuses predictive analytics and optimization solutions for commodity based supply chains. Prior to working at SignalDemand, Dr. Van Roo worked at the RAND Corporation with the United States military and United States government on strategic supply chain initiatives. He has several years of experience working as a supply chain consultant and strategic advisor to private and public firms in the United States. Dr. Van Roo holds a B.S. in Computer Science and Engineering, a Master’s in Business Administration, and a Ph.D. in Business from the University of Wisconsin-Madison.

**Christine Di Martinelly** is an assistant professor at IESEG School of Management in France. She teaches courses in production management and operations research. She works on optimization, risk, and performance management. Her main interests are the health care supply chain and logistics of the mine industry. Dr. Di Martinelly holds a management engineering degree from Catholic University of Mons (FUCAM), a master’s degree in applied statistics from Gembloux Agricultural University, and a doctorate in sciences from INSA Lyon (France).

**Fouad Riane** is a professor of production management at Louvain School of Management, Catholic University of Mons (FUCAM). He teaches graduate courses in production management, supply chain management, operations research, and simulation. He is the director of the Research Centre of Industrial Management (CREGI). He works on modeling, optimization, and simulation areas. His main research interests are production planning, supply chain optimization, and maintenance management. He has published several papers in well-established journals. Dr. Riane holds a mechanical engineering degree from Mohamedia School of Engineers (Morocco) and a management and commercial engineering degree and a doctorate in management science from FUCAM University.